Footprints: outline

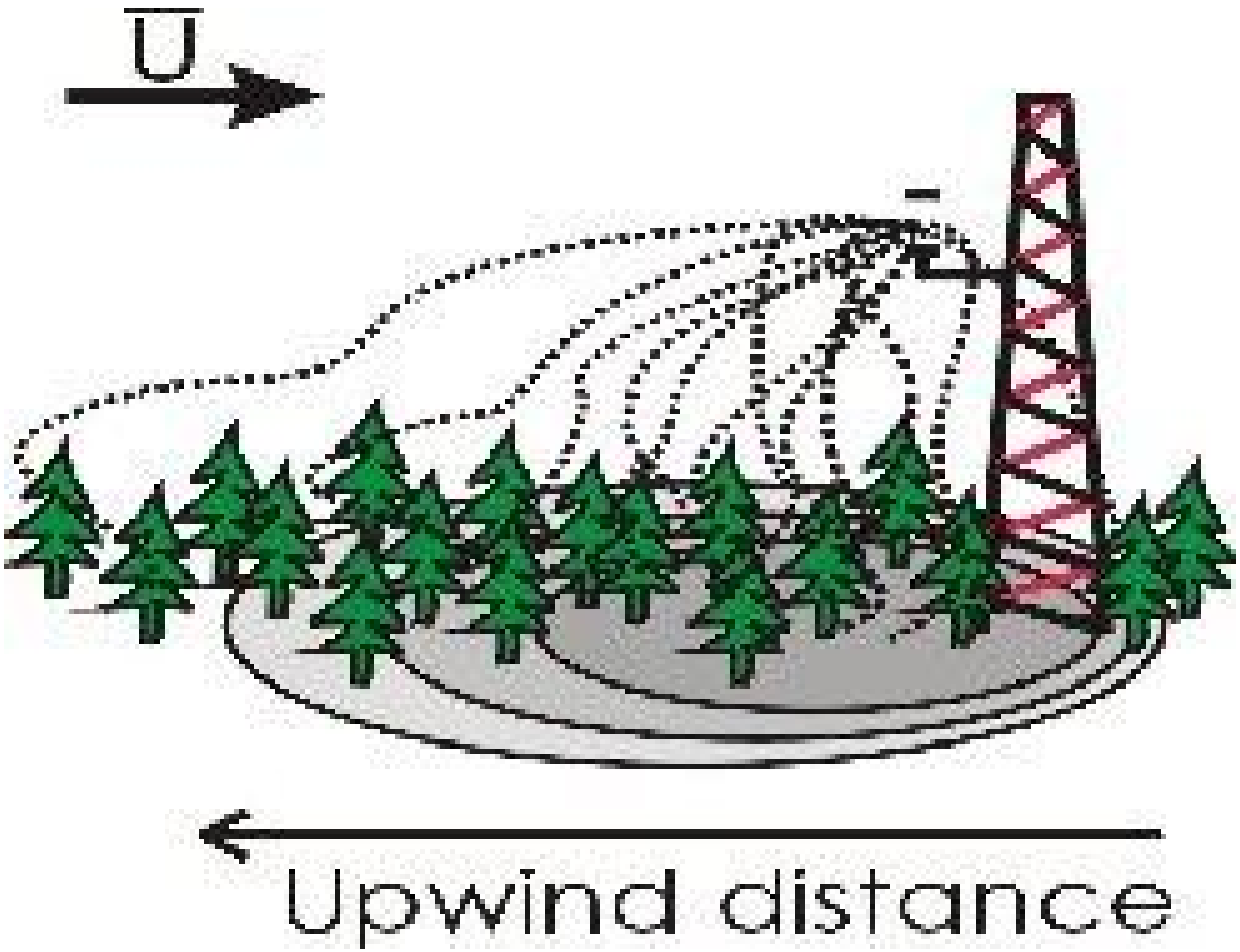
Üllar Rannik
University of Helsinki

-Concept of footprint and definitions
-Analytical footprint models
-Model by Korman and Meixner
-Footprints for fluxes vs. concentrations
-Footprints for gradient and profile techniques
-Footprint evaluation other methods
-Lagrangian stochastic models

-Vegatated canopies
-Inhomogeneos flows and complex terrain

- No too detailed derivations

“Field of view” of flux measurement



Footprint function (of flux or concentration)

The footprint function *f* is given by the integral equation of diffusion

$$\eta = \int_{\mathbb{R}} \phi(\vec{x}, \vec{x}')Q(\vec{x}')d\vec{x}$$

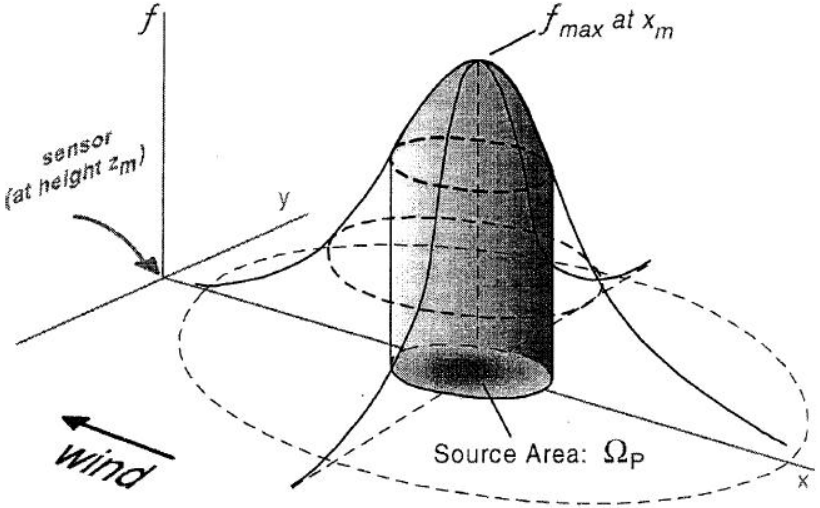
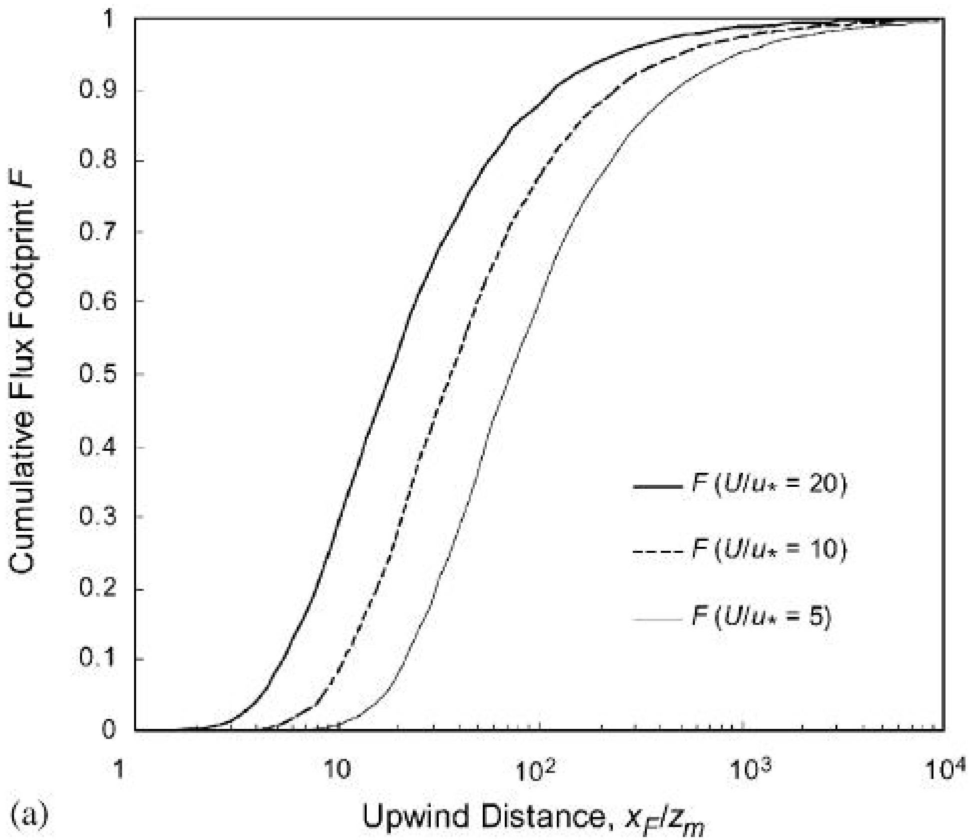


Fig. 1. Schematic of the source weight function, or footprint function. The source weight is small for small separation distances. It rises to a maximum with increasing distance and then falls off again to all sides as the separation is further increased (adapted from Schmid, 1994).

where η is the quantity being measured at location $\vec{x}$ (note that $\vec{x}$ is a vector) and $Q(\vec{x}')$ is the source emission rate/sink strength in the surface-vegetation volume $\mathbb{R}$. η can be the concentration or the vertical eddy flux and ϕ is then concentration or flux footprint function, respectively

Definitions (1)

- Effective fetch (Gash, 1986): x_F as the "*F*-fraction effective fetch", *F* is an integral footprint function that expresses the upstream-integrated source weight as the portion of a measured flux contributed by sources within a limited fetch, scaled by the total flux from sources in an unlimited fetch



(a)

Definitions (2)

- Footprint (Schmid, 2002): the term *footprint* is used to summarize the notions of effective fetch, source area or sensor footprints, but each of these terms will be defined more formally
- *Footprint function or the source weight function*: Functions describing the relationship between the spatial distribution of surface sources/sinks and a signal

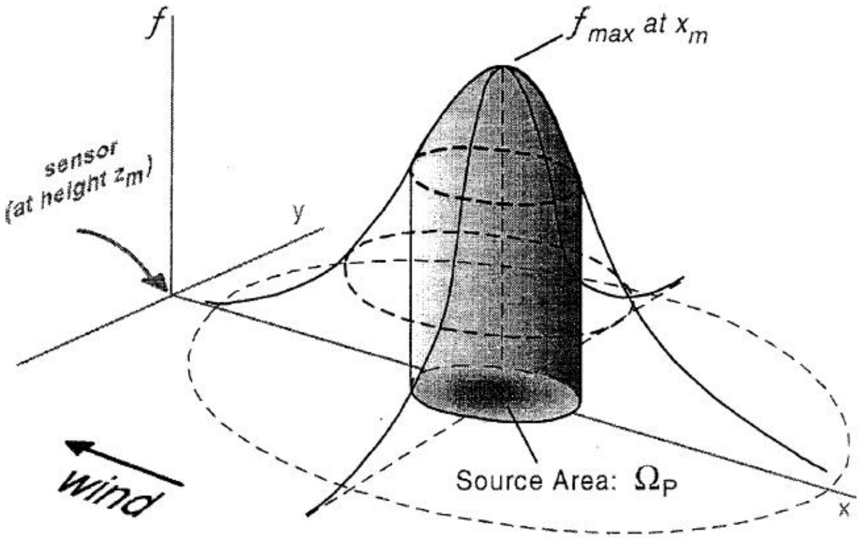


Fig. 1. Schematic of the source weight function, or footprint function. The source weight is small for small separation distances. It rises to a maximum with increasing distance and then falls off again to all sides as the separation is further increased (adapted from Schmid, 1994).

Definitions (3)

- Source area: Fraction of the surface (mostly upwind) containing effective sources and sinks contributing to a measurement point
- smallest possible area to be responsible for a given relative weight, *P*, (half, say *P* = 0.5) to the measured value and termed it the *source area* of level *P*. Source area, Ω_P, is bounded by a footprint isopleth $f(x, y, z_m) = f_P$ such that *P* is the fraction of the total integrated footprint function contained in the source area

$$P = \frac{\varphi P}{\varphi_{\mathrm{tot}}} = \frac{\iint_{\Omega_P} f(x', y', z_{\mathrm{m}})\,\mathrm{d}x'\,\mathrm{d}y'}{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x', y', z_{\mathrm{m}})\,\mathrm{d}x'\,\mathrm{d}y'}$$

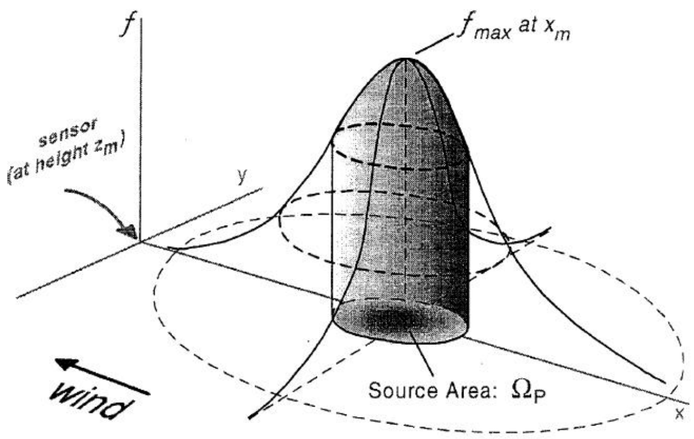


Fig. 1. Schematic of the source weight function, or footprint function. The source weight is small for small separation distances. It rises to a maximum with increasing distance and then falls off again to all sides as the separation is further increased (adapted from Schmid, 1994).

Analytical models (1)

Schuepp et al. (1990)

•Neutral stratification

•Constant wind speed profile

Horst and Weil (1992) - widely used (together with its successors)

•Flux footprint in cross-wind direction proportional to crosswind concentration distribution

$$\bar{f}^y(x, z_m) = -\int_{z_0}^{z_m} \bar{u}(z) \frac{\partial}{\partial x} \bar{C}^y(x, z)\, dz \quad (9)$$

where the crosswind integrated concentration, $\bar{C}^y$ was given by van Ulden (1978) as

$$\bar{C}^y(x, z) = \frac{A}{U\bar{z}} \exp\left\{-\left(\frac{z}{b\bar{z}}\right)^r\right\} \quad (10)$$

A, b functions of parameter r
p ~ 1.55, taken as constant

Using K-theory, van Ulden (1978) expressed the growth rate of $\bar{z}(x)$ as

$$\frac{d\bar{z}}{dx} = \frac{K(p\bar{z})}{\bar{u}(p\bar{z})p\bar{z}} \quad (12)$$

where K is the eddy diffusivity, and p is a (weak) function of r.

$$\bar{z}(x) = \frac{\int_0^\infty z\bar{C}^y(x, z)\, dz}{\int_0^\infty z\bar{C}^y(x, z)\, dz}$$

Analytical models (2)

- Model by H&W (1992) not fully analytical, requires numerical integration
- H&W 1994 presented an approximate analytical solution (exact for power law wind profiles)

$$\bar{f}^y \simeq \frac{d\bar{z}}{dx}\frac{z_m}{\bar{z}^2}\frac{\bar{u}(z_m)}{\bar{u}(c\bar{z})}A\,e^{-(z_m/b\bar{z})^r}$$

The exponent r is discussed in the last paragraph of this section. $\bar{z}$ is calculated from the Lagrangian similarity formula of van Ulden (1978)

$$\frac{d\bar{z}}{dx} = \frac{k^2}{[\ln(p\bar{z}/z_0) - \psi_m(p\bar{z}/L)]\phi_h(p\bar{z}/L)}. \quad (6)$$

However, Gryning et al. (1983) proposed a formula for r that is a continuous function of atmospheric stability and which is consistent with the assumptions that are the basis for (6),

$$r = 1 + \frac{c\bar{z}}{\phi_h(c\bar{z}/L)}\left[\frac{\partial\phi_h}{\partial z}\right]_{c\bar{z}} + \frac{\phi_m(c\bar{z}/L)}{\ln(c\bar{z}/z_0) - \psi_m(c\bar{z}/L)}. \quad (8)$$

AN ANALYTICAL FOOTPRINTMODEL FOR NON-NEUTRAL STRATIFICATION by ROBERT KORMANN and FRANZ X. MEIXNER

Boundary-Layer Meteorology 99: 207–224, 2001

- Stationary gradient-diffusion formulation (as other models)
- Heigh-independent cross-wind dispersion (as other models)
- Power law profiles of the mean wind velocity U and the eddy diffusivity K (as other models)
- Power-law solution of the two-dimensional advection-diffusion equation (PLS of ADE)
- Horst and Weil (1992, 1994) reintroduce the Monin-Obukhov similarity profiles into the PLS of ADE and derive the solution for footprint function -> infringement of the continuity equation.
- Kormann and Meixner derive the final solution for footprint function by keeping the power law profiles. Only then perform fitting of power laws (K, U) to M-O similarity profiles

Footprint model Korman and Meixner (2)

Two-dimensional advection-diffusion equation
c, f – here cross-wind integrated concentration and flux

$$u\frac{\partial c}{\partial x} = -\frac{\partial f}{\partial z}.$$

Power law profiles for wind speed and eddy diffusivity

$$u(z) = Uz^m \quad \text{and} \quad K(z) = \kappa z^n$$

By using cross-wind integrated concentration c and flux or footprint f (per unit point source), where D_z is the vertical distribution function of concentration due to point source

$$f(x,z) = -\frac{K}{\overline{u}}\frac{\partial D_z}{\partial z} = -K\frac{\partial c(x,z)}{\partial z},$$

Solution to advection-diffusion equation

$$c(x,z) = \frac{A}{\overline{z}\,\overline{u}}\exp\left[-\left(\frac{Bz}{\overline{z}}\right)^r\right]$$

With A and B as follows and the shape factor $r = 2+m-n$

$$A = \frac{r\Gamma(2/r)}{\Gamma(1/r)^2} \text{ and } B = \frac{\Gamma(2/r)}{\Gamma(1/r)},$$

Footprint model Korman and Meixner (3)

Using the solution for cross-wind integrated concentration

$$c(x,z) = \frac{A}{\overline{z}\,\overline{u}} \exp\left[-\left(\frac{Bz}{\overline{z}}\right)^r\right]$$

and calculating the derivative of the mean plume height, it follows

$$\frac{\mathrm{d}\overline{z}}{\mathrm{d}x} = r\,B^r\,\frac{\kappa}{U}\overline{z}^{1-r}$$

<u>Here</u>

•Horst and Weil express the gradient of the mean plume height in terms of power law profiles (and power law dependence disappears)

•Korman and Meixner integrate the gradient of the mean plume height without such replacement (and retain power law representation) and obtain

$$\overline{z}(x) = B\left(\frac{r^2\kappa}{U}\right)^{1/r} x^{1/r}$$

From $c(x,z) = \frac{A}{\overline{z}\,\overline{u}} \exp\left[-\left(\frac{Bz}{\overline{z}}\right)^r\right]$ follows $c = \frac{1}{\Gamma(\mu)}\frac{r}{U z^{1+m}}\frac{\xi^\mu}{x^\mu}e^{-\xi/x}$

Footprint model Korman and Meixner (4)

And further from the definition
$$f(x,z) = -\frac{K}{\overline{u}}\frac{\partial D_z}{\partial z} = -K\frac{\partial c(x,z)}{\partial z},$$

the expression for cross-wind integrated footprint function
$$f = \frac{1}{\Gamma(\mu)}\frac{\xi^{\mu}}{x^{1+\mu}}e^{-\xi/x}$$

Which defines also the (cross-wind distributed) footprint function via
$$\phi(x,y,z) = D_y f$$

where $\mu = (1+m)/r$ and $\xi(z) = \frac{U z^r}{r^2 \kappa}$

Only now Korman and Meixner relate the power law profiles to M-O similarity profiles by proposing

- Analytical solution relating power law expressions to M-O expressions by using heigh $z = z_m$
- Numerical approach by minimizing expressions by using $z_1 = 3\, z_0$ and $z_2 = (1+k)\, z_m$.

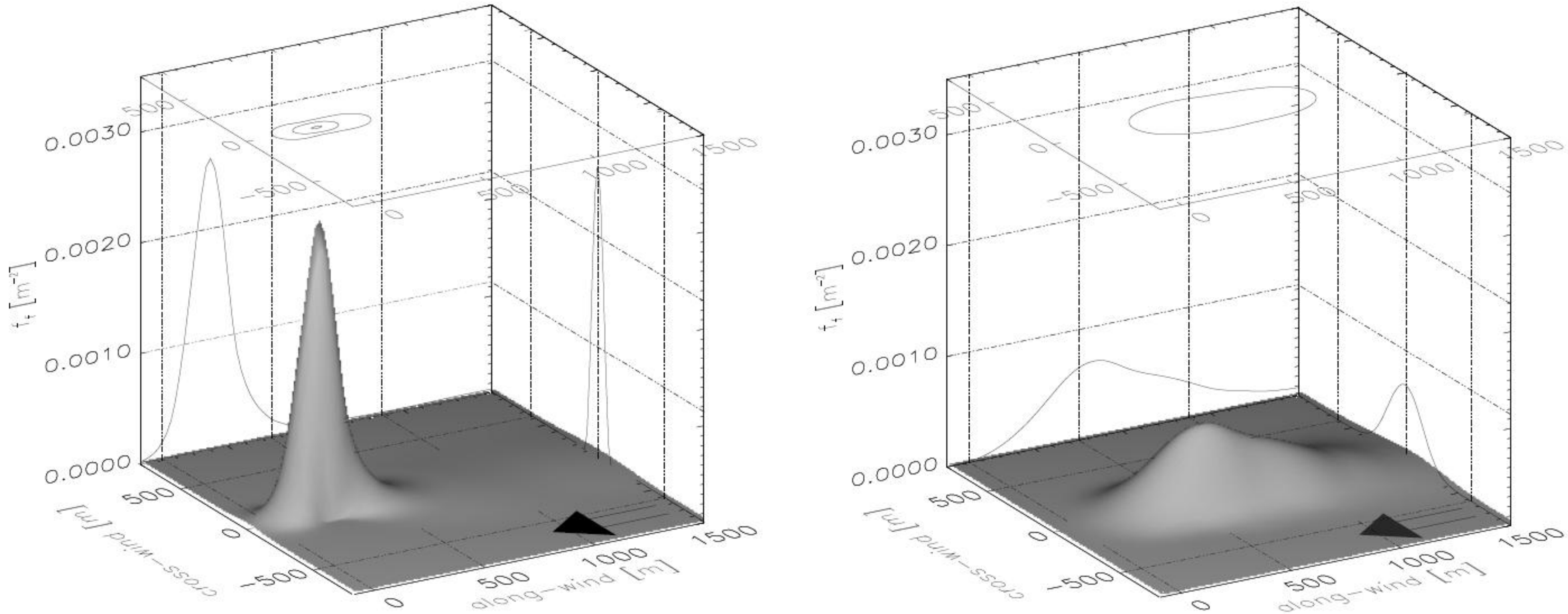
to obtain m and n and proportionality coefficient U and k

$$\epsilon_u^2(m,U) = \int_{z_1}^{z_2}(u-\widetilde{u})^2\,dz \bigg/ \int_{z_1}^{z_2} u^2\,dz$$

and

$$\epsilon_K^2(n,\kappa) = \int_{z_1}^{z_2}(K-\widetilde{K})^2\,dz \bigg/ \int_{z_1}^{z_2} K^2\,dz.$$

Footprint functions

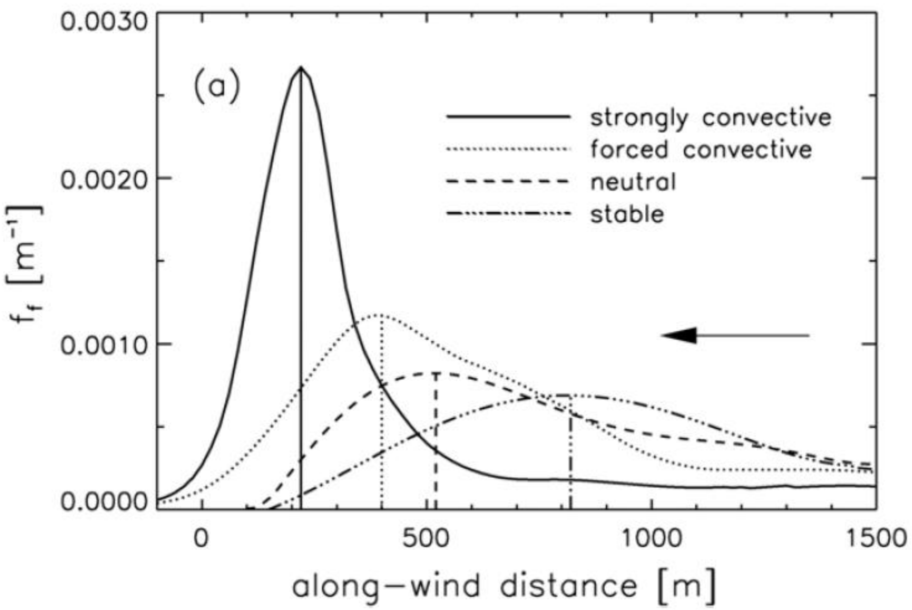


strongly convective stable

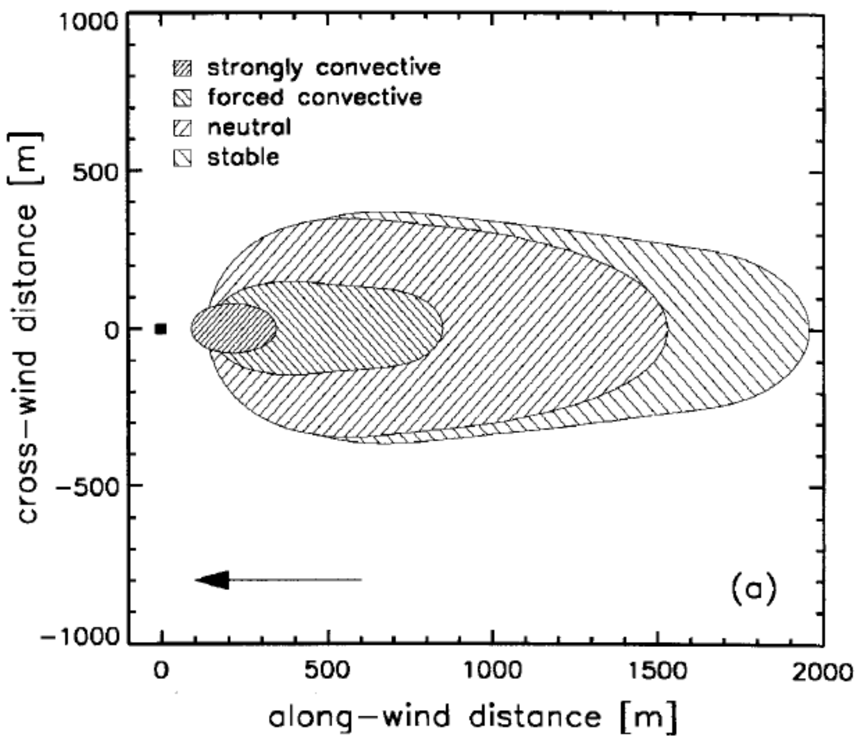
Cross-wind integrated footprint function and source areas (Kljun et al. 2002)

	u_* [m/s]	w_* [m/s]	L [m]	z_i [m]
strongly convective	0.2	2.0	-5	2000
forced convective	0.2	1.0	-36	1800
neutral	0.8	–	∞	1500
stable	0.3	–	300	300

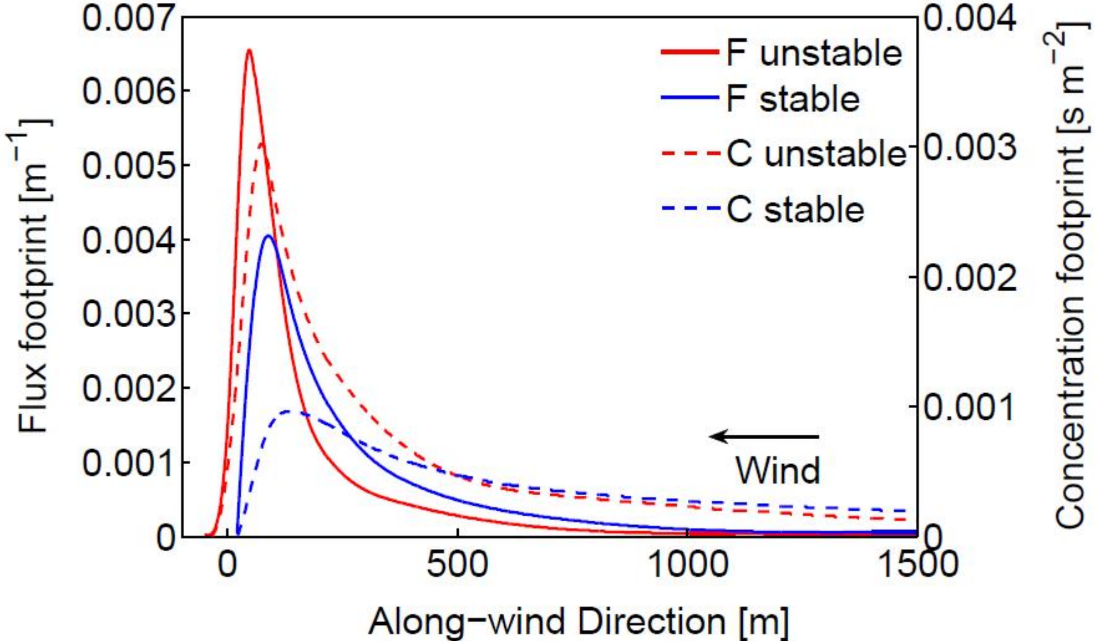
Receptor location at (0,0,50)m

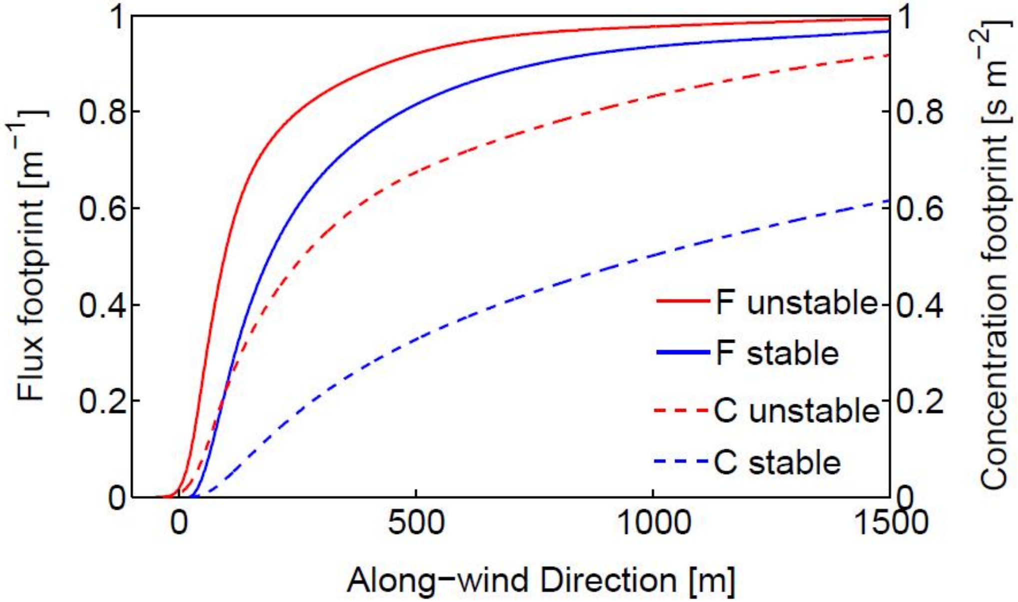


50% level source area



Concentration vs. flux footprints (1)





Concentration vs. flux footprints – source area (2)

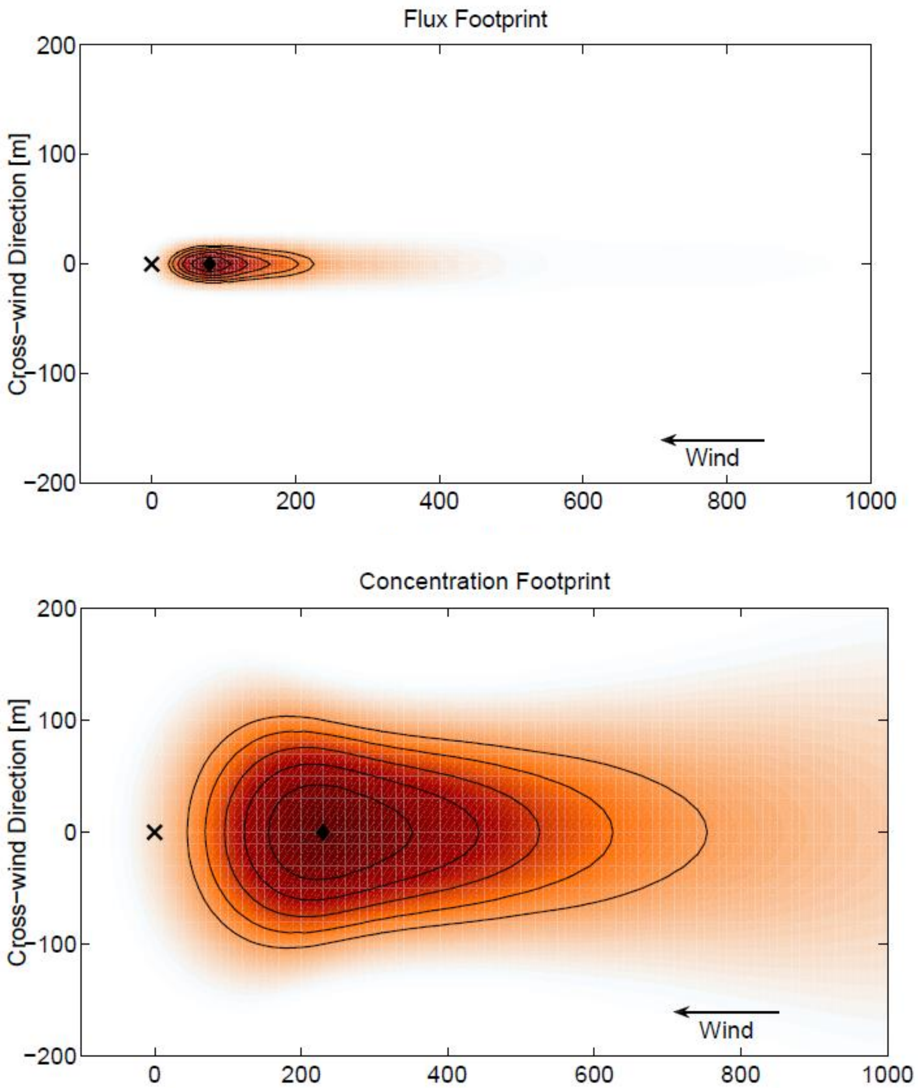
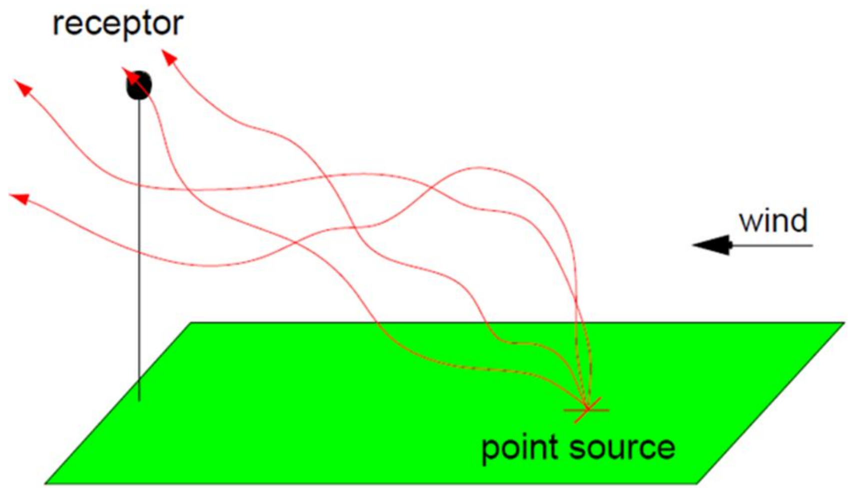


Fig. 8.3 Footprint functions for neutral atmospheric stratification conditions ($u_* = 0.8\ \mathrm{m\,s^{-1}}$, $z_i = 1{,}500$ m) at 10 m height and 0.01 m roughness length for (**a**) flux and (**b**) concentration. The *isolines* represent 10–50% source area. *Cross* denotes the tower location

Concentration vs. flux footprints (3) – qualitative view

- Surface emissions contribute to flux proportionally with direction of movement of air parcel.
- Close to measurement point emissions move predominantly upwards, contributing to flux footprint with the same sign.
- Far from the measurement point, the number of upward and downward movements of particles or fluid elements is more balanced
- Each air parcel contributes positively to the concentration footprint independently of the direction of the trajectory. This increases the footprint value at distances further apart from the receptor location.



Analytical models: gradient method
Horst 1999, BLM 90, 171-188.

Flux-gradient relationship defined as

$$\phi_h(z/L) = \frac{kz}{c_*}\frac{\partial C}{\partial z},$$

The cross-wind integrated flux follows from cross-wind integrated concentration profile

$$\bar{F}^y(z) = -\frac{u_* kz}{\phi_h(z/L)}\frac{\partial \bar{C}^y}{\partial z}. \qquad (12)$$

The atmospheric concentration downwind of a surface area source can again be calculated by superposition (e.g., Horst, 1978; Schmid, 1994),

$$\bar{C}^y(x,z) = \int_0^\infty \bar{F}_0^y(x-s)\bar{D}^y(s,z)\,\mathrm{d}s, \qquad (10)$$

HW92 used an analytic model for the crosswind-integrated concentration distribution within the atmospheric surface-flux layer,

$$\bar{D}^y(x,z) = \frac{A}{\bar{z}U}\mathrm{e}^{-(z/b\bar{z})^r}, \qquad (3)$$

Substituting (10) into (12), we obtain an equation that defines $\bar{f}_g^y$, the crosswind-integrated flux footprint for a gradient flux measurement

$$\begin{aligned}\bar{F}^y(x,z_m) &= -\frac{u_* k z_m}{\phi_h(z_m/L)}\int_0^\infty \bar{F}_0^y(x-s)\left.\frac{\partial \bar{D}^y(s,z)}{\partial z}\right|_{z=z_m}\mathrm{d}s \\ &= \int_0^\infty \bar{F}_0^y(x-s)\bar{f}_g^y(s,z_m)\,\mathrm{d}s, \qquad (13)\end{aligned}$$

Analytical models: grad and profile method

The cross-wind integrated footprint for gradient technique

$$\bar{f}_g^y = -\frac{u_* k z_m}{\phi_h(z_m/L)} \frac{\partial \bar{D}^y(x,z)}{\partial z}\bigg|_{z=z_m} = \frac{A u_* k r}{\bar{z} U \phi_h(z_m/L)} \left(\frac{z_m}{b\bar{z}}\right)^r \mathrm{e}^{-(z_m/b\bar{z})^r}. \qquad (14)$$

Vertical concentration gradients are most commonly estimated by measuring vertical concentration profiles over finite height intervals. For the simple case of concentration measurements at two heights, z_1 and z_2, the finite-difference flux-profile relationship is found by vertical integration of (12) using the assumption that the flux is independent of height,

$$\bar{F}^y = \frac{-u_* k[\bar{C}^y(z_2) - \bar{C}^y(z_1)]}{\ln(z_2/z_1) - \psi_h(z_2/L) + \psi_h(z_1/L)}, \qquad (15)$$

where ψ_h describes the dependence of the scalar concentration profile on atmospheric stability. Substituting (10) into (15) we obtain an equation that defines $\bar{f}_d^y$, the crosswind-integrated footprint for a flux estimated from a two-level profile. Using (3) for $\bar{D}^y$ we then find

$$\bar{f}_d^y = \frac{-A u_* k}{\bar{z} U} \frac{\mathrm{e}^{-(z_2/b\bar{z})^r} - \mathrm{e}^{-(z_1/b\bar{z})^r}}{\ln(z_2/z_1) - \psi_h(z_2/L) + \psi_h(z_1/L)}. \qquad (16)$$

Analytical models: gradient & profile method

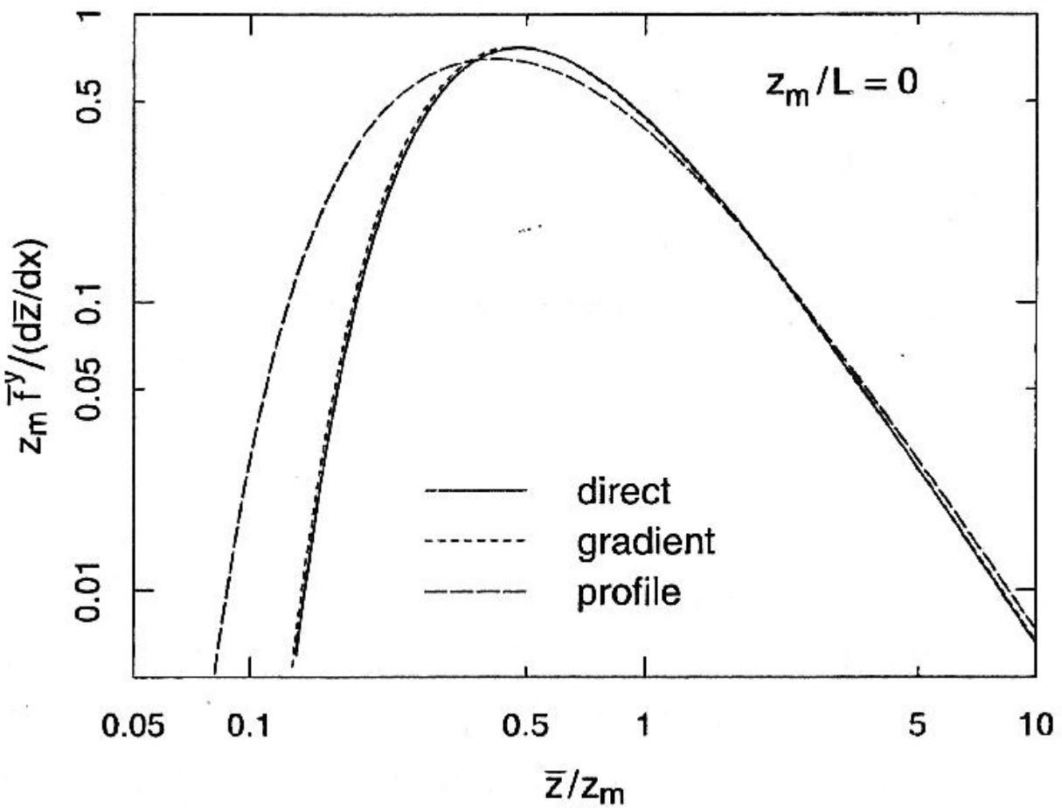


Figure 2. Normalized flux footprints as a function of $\bar{z}/z_m$ for $z_m/z_0 = 300$ (b) $z_m/L = 0$, (c) $z_m/L = 0.3$.

For profile method
$Z_2/Z_1 = 4$

Footprint for concentration profile flux estimates is similar to that of the footprint for eddy covariance measurements if the EC measurements are done at the arithmetic (stable) or geometric (unstable) mean height of the highest and lowest levels

Analytical models summary (6)

- assume a horizontally homogeneous turbulence field
- assumption of steady-state conditions during the course of the flux period analyzed
- Based on analytical solution to advection-diffusion equation
- Based on the Monin-Obukhov similarity theory (being also applied to the layer of air above the tower)
- assume that no contribution to a point flux is possible by downwind sources (no along-wind turbulent diffusion)
- unable to include the influence of non-local forcings to flux measurements
- no along-wind diffusion included

Other types of footprint models

- Analytical models
  - Applicable to Atmospheric Surface Layer parameterisation only
- Lagrangian Stochastic models
  - Require pre-defined turbulence field
  - Can be applied to Atmospheric Boundary Layer (ABL) + vegetation canopy + inhomogeneous flow + complex terrain
- Closure model based
  - Capable to simulate turbulence field
  - Applicable to vegetation canopy + inhomogeneous flow + complex terrain
  - Can be combined with LS model
  - Numerically demanding
- Large Eddy Simulation based
  - Most advanced model
  - Capable to simulate turbulence field
  - Applicable to ABL + vegetation canopy + inhomogeneous flow (+ terrain with limited complexity)
  - Can be combined with LS model
  - Numerically very demanding

Lagrangian Stochastic models (1)

Consists of

- a trajectory simulation model
- Estimator for footprint function

Diffusion of a scalar described by means of a stochastic differential equation, a generalized Langevin equation

$$d\mathbf{X}(t) = \mathbf{V}(t)dt$$

$$d\mathbf{V}(t) = \mathbf{a}(t, \mathbf{X}(t), \mathbf{V}(t))dt + \sqrt{C_0 \bar{\varepsilon}(\mathbf{X}(t), t)}d\mathbf{W}(t)$$

where *X(t)* and *V(t)* denote trajectory co-ordinates and velocity as a function of time *t*, C_0 is the Kolmogorov constant, is the mean dissipation rate of turbulent kinetic energy and *W(t)* describes the 3-dimensional Wiener process.

$\mathbf{a}(t, \mathbf{X}(t), \mathbf{V}(t))$ to be defined for each particular LS model

<u>NB! Lagrangian Stochastic models not uniquely defined for more than 1D atmospheric flow</u>

Lagrangian Stochastic models (2)

Inputs required by LS trajectory model (Gaussian)

•Mean wind speed

•Variances of wind speed components

•Momentum flux

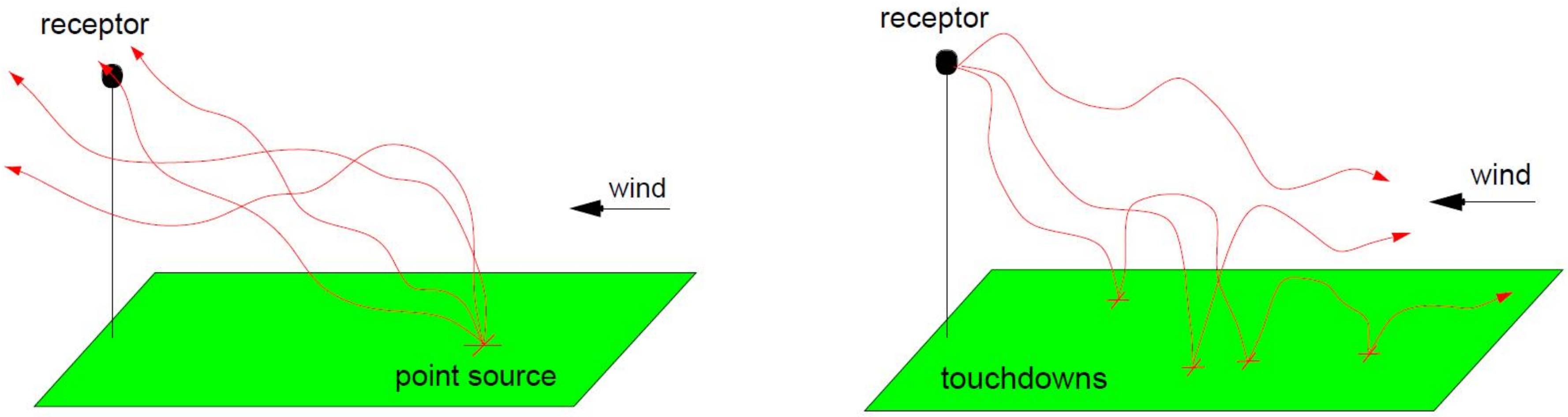
•Dissipation rate of TKE

In addition for non-Gaussian

•Skewness of wind speed components (third moments)

•Kurtosis of wind speed components (fourth moments)

Forward vs. backward time frame (3)

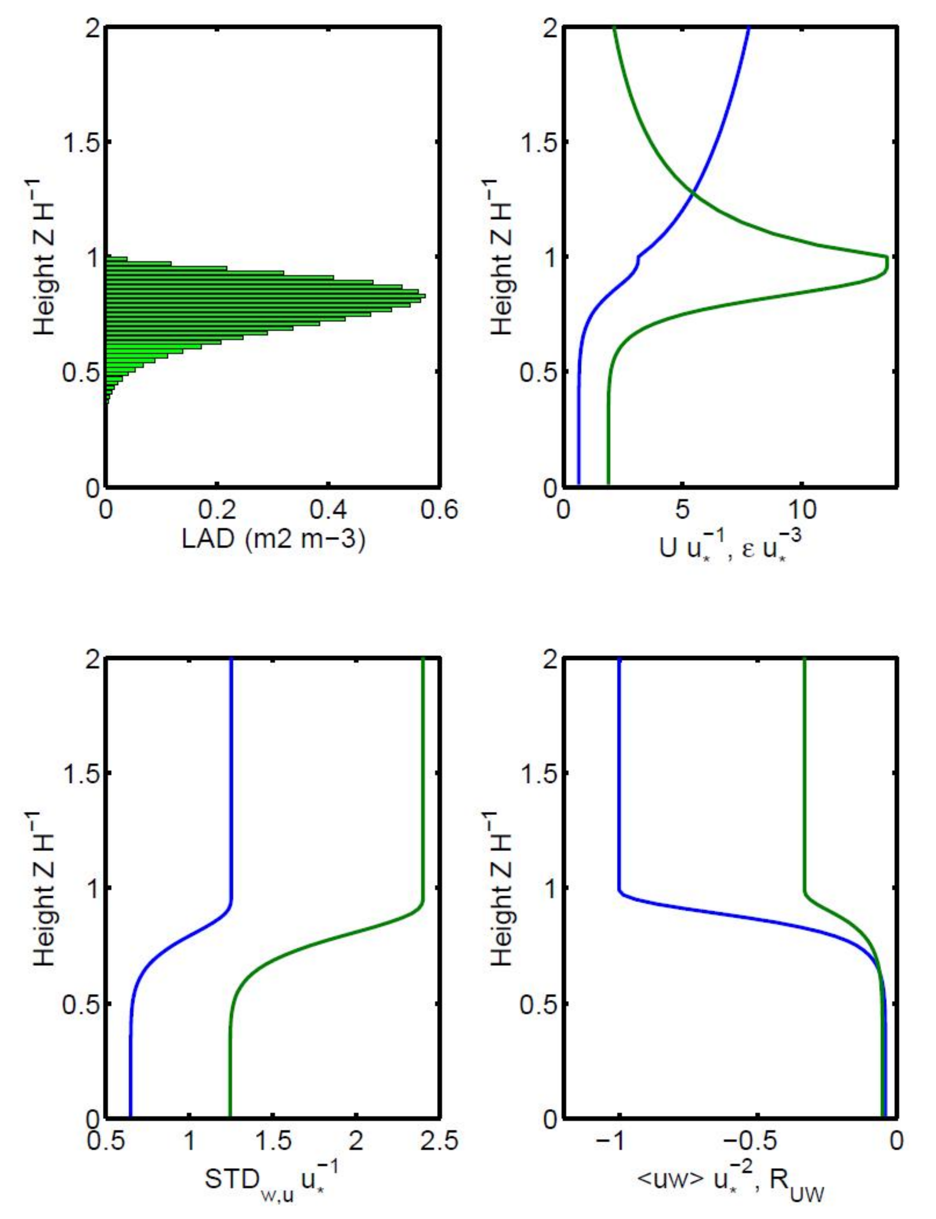


Algorithm: integration of LS trajectories and calculation of statistics

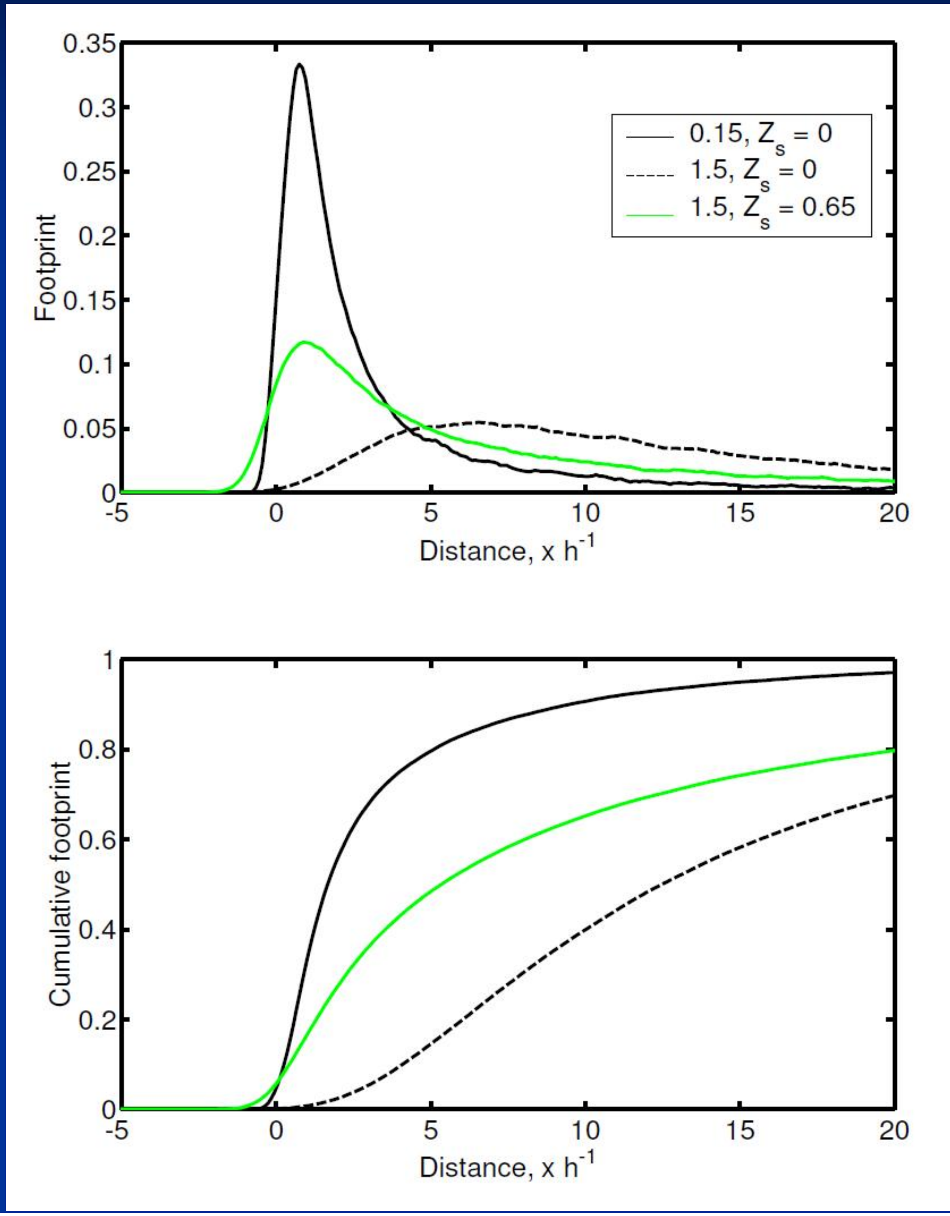
a)at interception with observation level

b)at particle “touch-down”

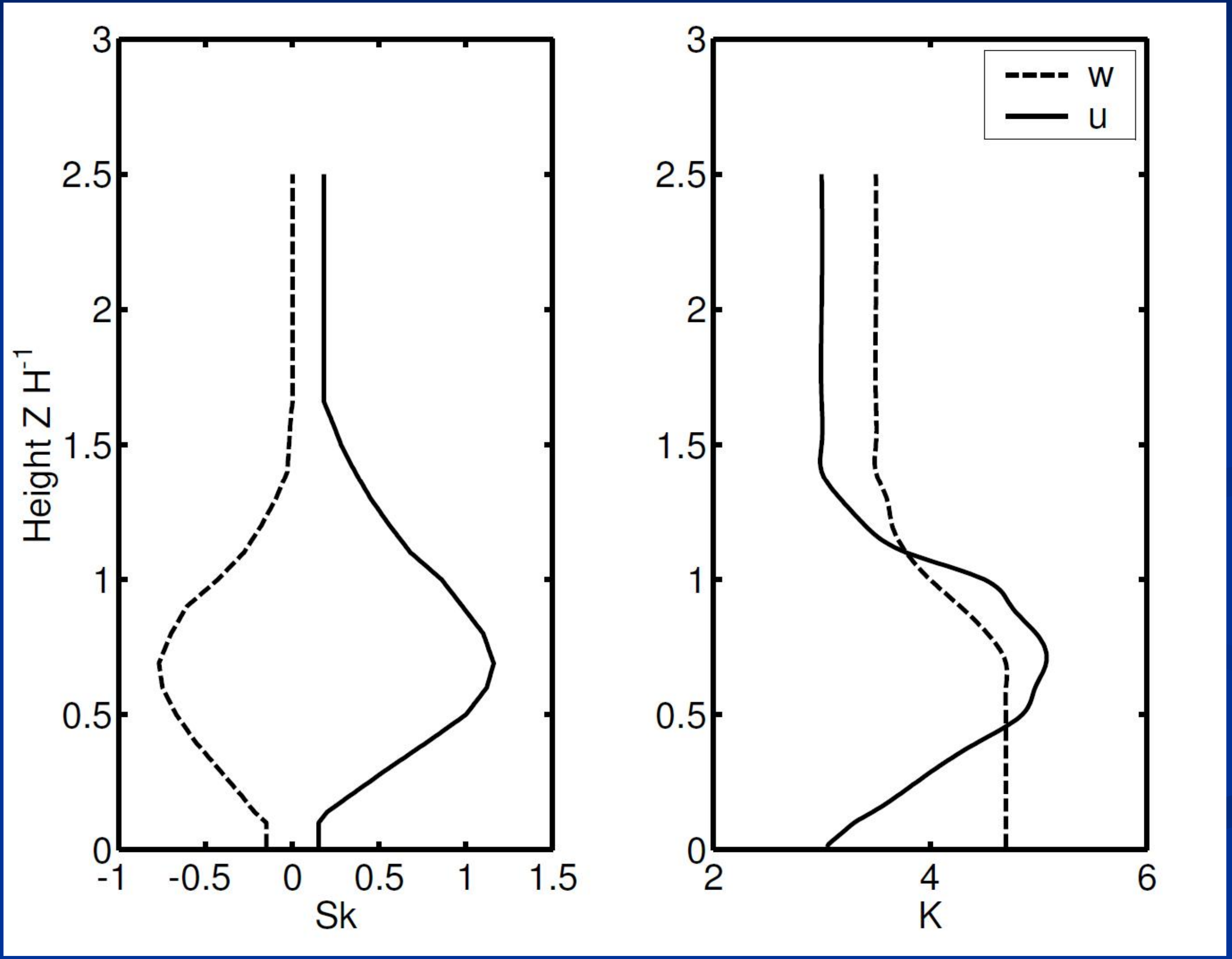
Footprints & high vegetation (1)



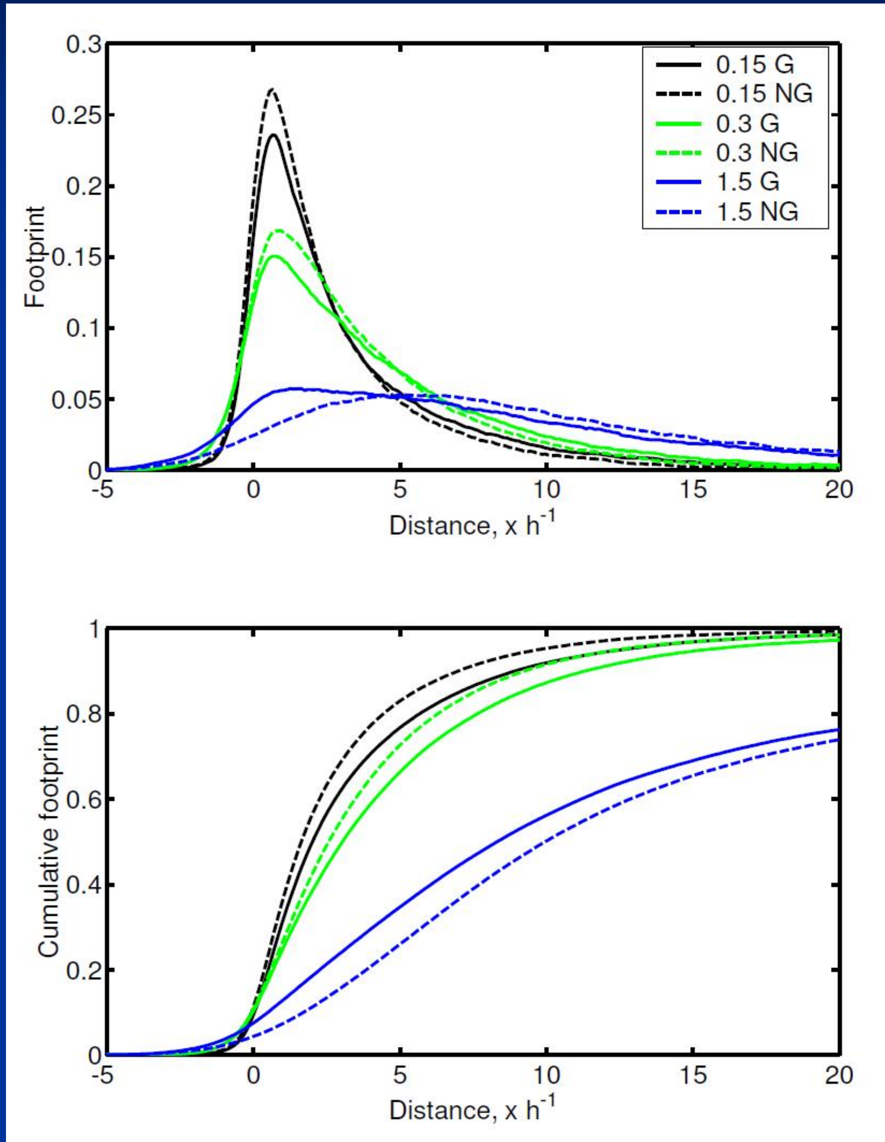
Footprints inside and over forest (2)



Non-Gaussian turbulence statistics inside canopy (3)



Flux footprint pdf with Gaussian (G) and non-Gaussian (NG) turbulence profiles (4)

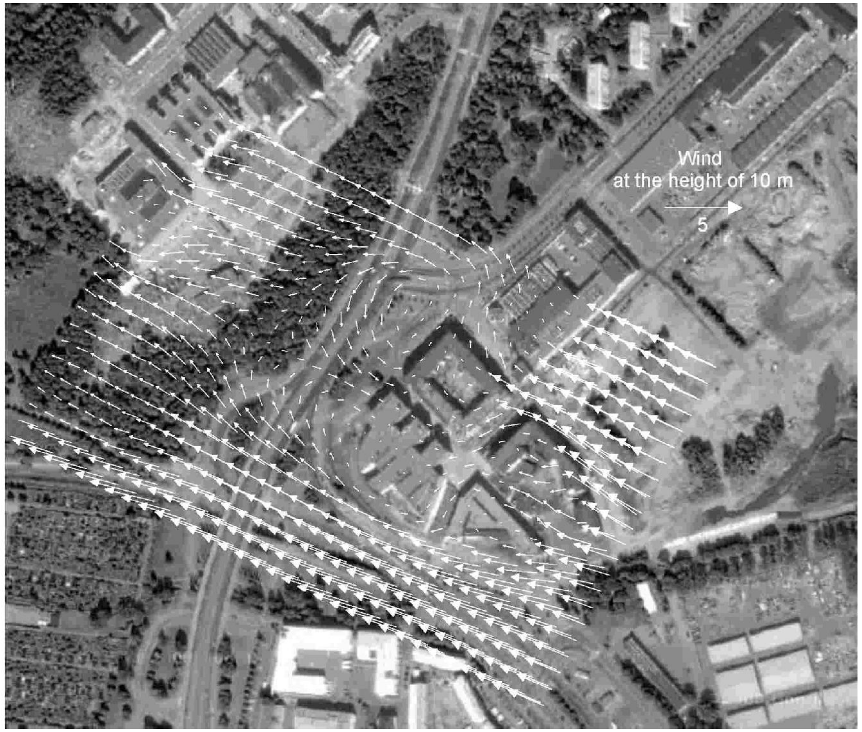


Inhomogeneous flow and complex terrain (1)

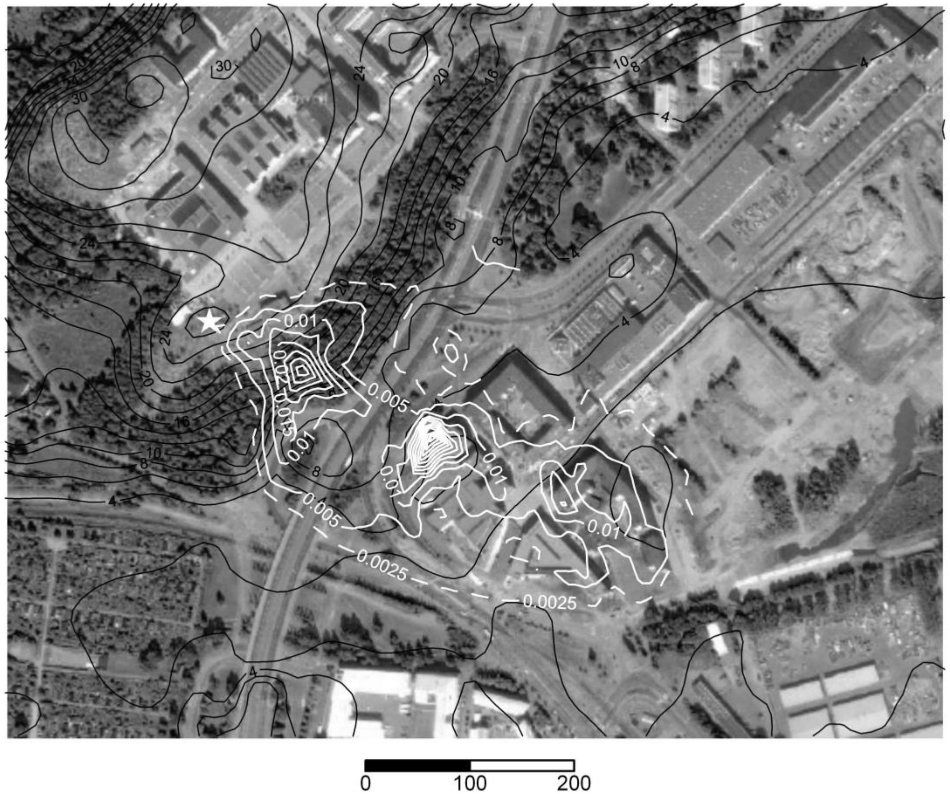
- Forest canopy inhomogeneity
- Complex terrain + vegetation
- Urban environment

Urban area (4)

wind vector

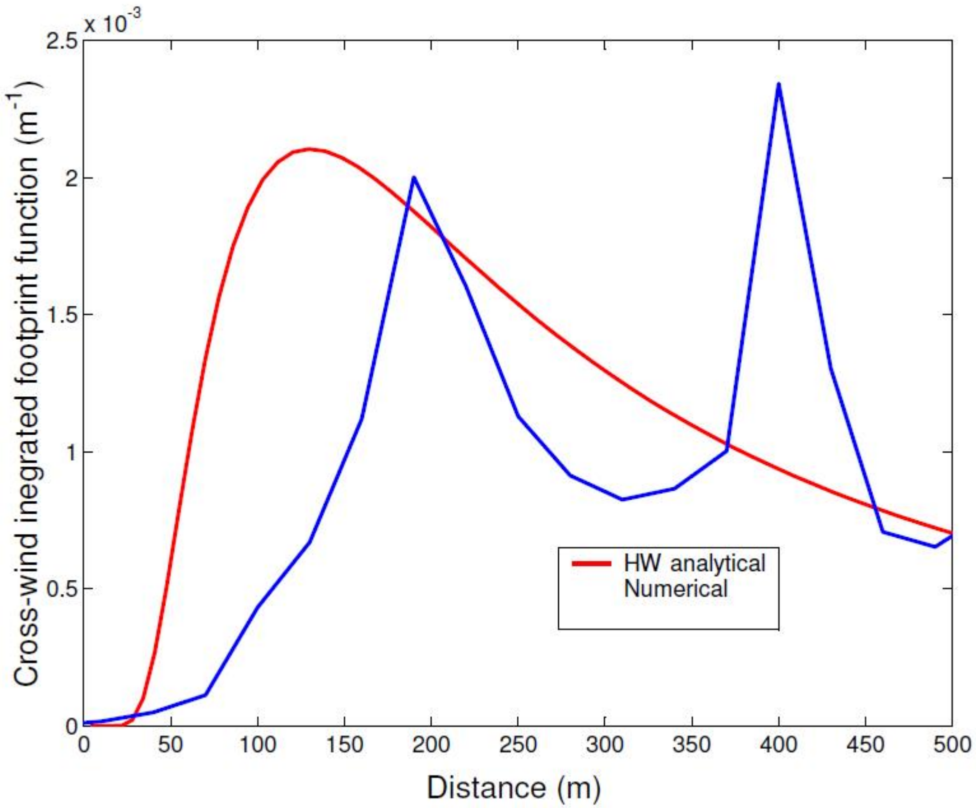


footprint



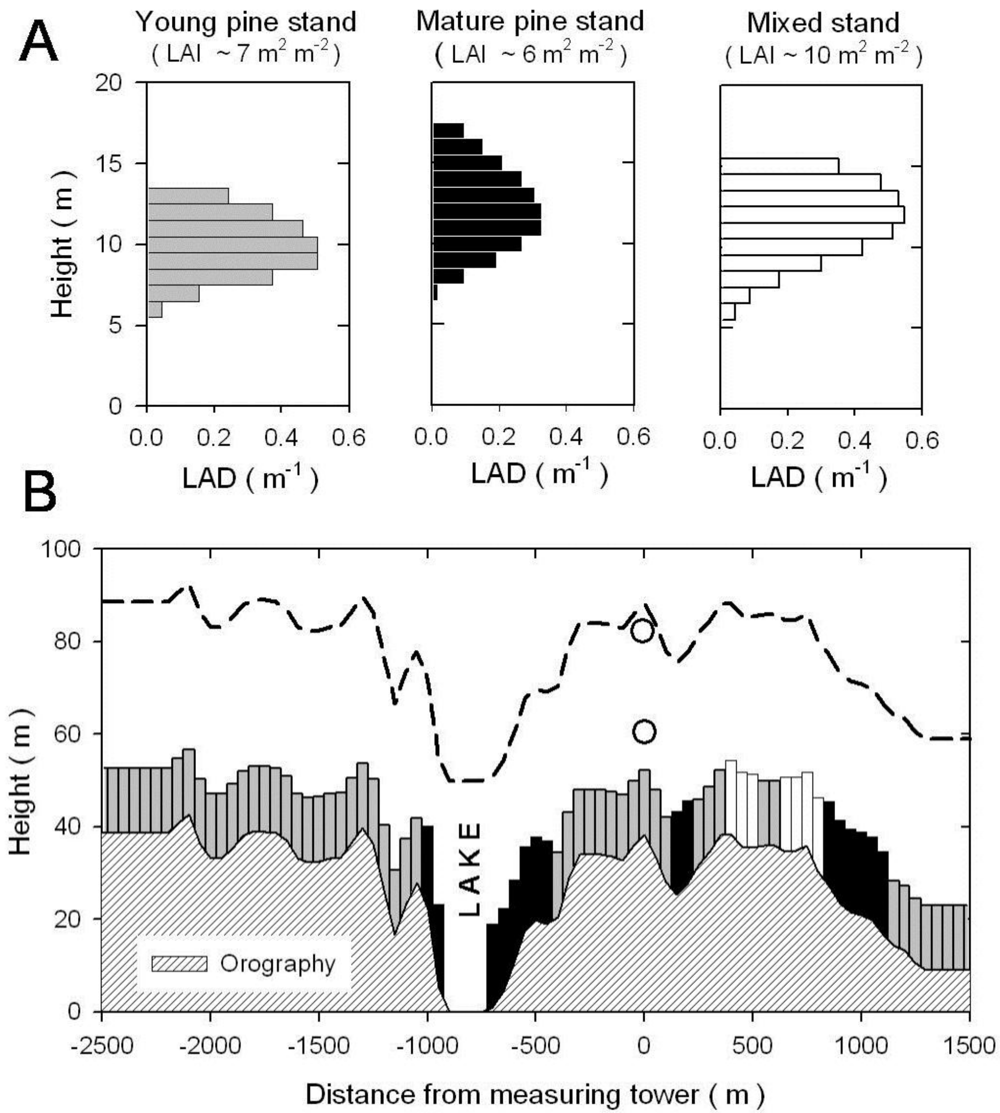
Aerial photograph of the measurement location. Topography of the measurement site (relative to sea level) is denoted by black contours. Vector plots (a) and the flux footprint function (b) (scale 10^{-6}, the unit of flux footprint is m^{-2}) are shown when the wind direction is perpendicular to the road (117^o), Geostrophic wind speed is $10ms^{-1}$ and the boundary layer is neutrally stratified. The location of the measurement tower is marked by a white star, and its distance to the edge of the road is around 150 m. (After Jarvi et al., 2009)

Urban area (5)



Cross-wind integrated flux footprint as estimated for surface sources for flux measurements from road direction (wind direction 117°). Neutral stratification was assumed. Measurement height H = 31 m. Footprint function as obtained from analytical (Horst and Weil, 1994) and numerical model predictions. After Järvi et al. (2009)

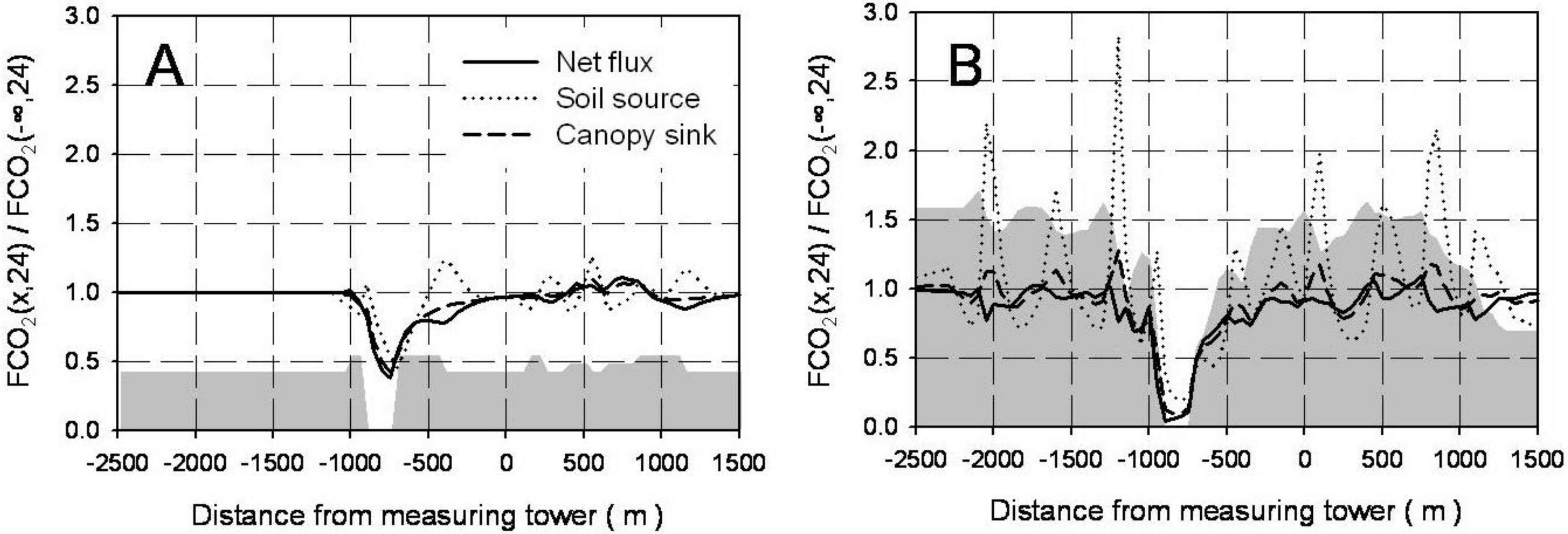
Complex terrain + inhomogeneous canopy (2)



Stand structures (A) and its distribution along west-east transect together with topography variation (B) used in the simulation for Hyytiälä site.

Distance is counted from flux measurement tower; a narrow oblong lake is located at about 750 m to the west.

Complex terrain + inhomogeneous canopy (3)

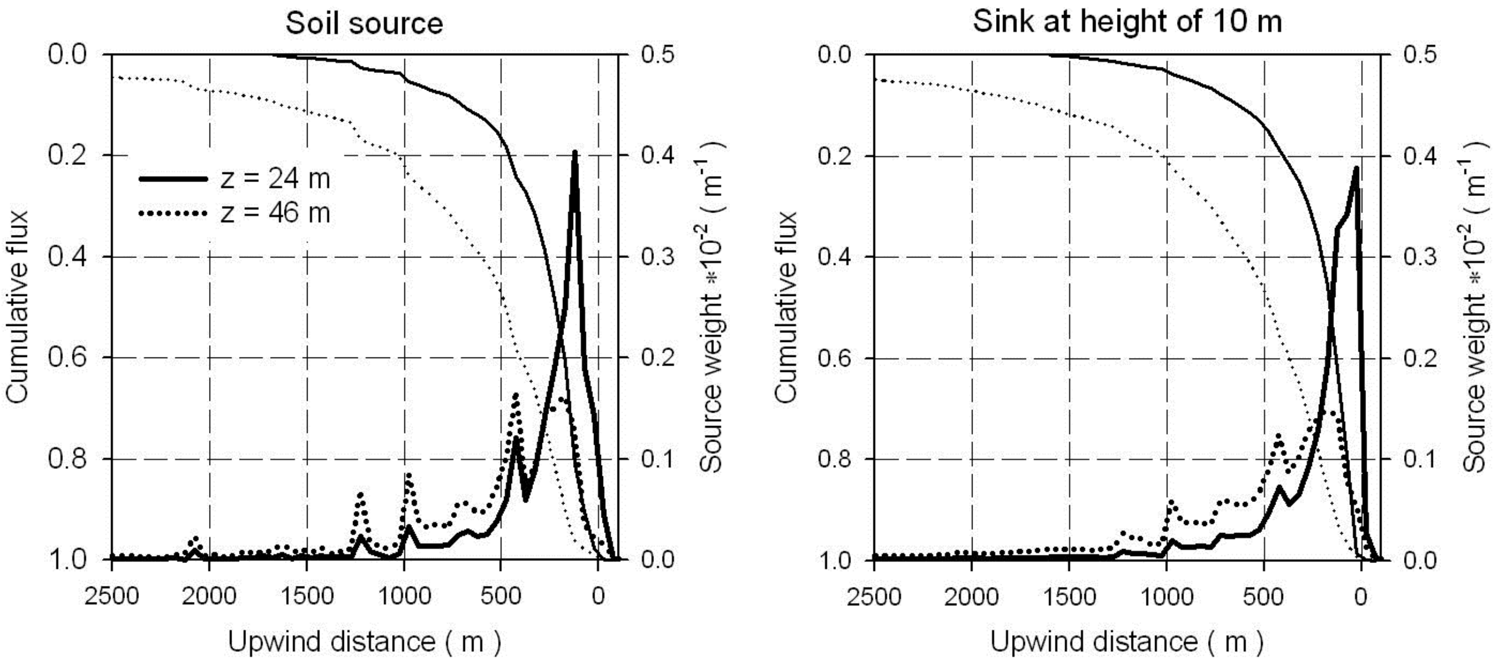


Normalised vertical CO_2 fluxes in case of west-to east neutral airflow at Hyytiälä site. The fluxes at heights 24 were normalised with far upwind values at the same height.

A- Influence of heterogeneous vegetation only

B - Together with topography variations.

Complex terrain + inhomogeneous canopy (4)



Footprint functions and their cumulative values for two heights at Hyytiälä measurement tower. Constant surface emission at ground surface and canopy uptake at effective level 10 m were considered. The figure shows an influence of varying topography on footprint function.

Milestones in footprint concept development

Author	Remarks
Pasquill (1972)	First model description, concept of effective fetch
Gash (1986)	Neutral stratification, concept of cumulative fetch
Schuepp *et al.* (1990)	Use of source areas, but neutral stratification and averaged wind velocity
Leclerc and Thurtell (1990)	Lagrangian footprint model
Horst and Weil (1992)	1-dimensional footprint model
Schmid (1994), (1997)	Separation of footprints for scalars and fluxes
Leclerc *et al.* (1997)	LES model for footprints
Baldocchi (1997)	Footprint model within forests
Rannik *et al.* (2000; 2003)	Lagrangian model for forests
Kormann and Meixner (2001)	Analytical model with exponential wind profile
Kljun *et al.* (2002)	Three dimensional Lagrangian model for various turbulence stratifications with backward trajectories
Sogachev and Lloyd (2004)	Boundary-layer model with 1.5 order closure
Sogachev *et al.* (2004)	Footprint estimates for a non-flat topography
Strong *et al.* (2004)	Footprint model with reactive chemical compounds
Cai and Leclerc (2007)	Footprints from backward and forward in-time particle simulations driven with LES data
Klaassen and Sogachev (2006)	Footprint estimates for a forest edge
Vesala *et al.* (2008a)	Footprint estimates for a complex urban surface
Steinfeld *et al.* (2008)	Footprint model with LES embedded particles
Castellvi (2012)	Footprints for surface renewal method